## Probability Cheat Sheet

## Calculating probabilities

An experiment is a repeatable process that gives rise to a number of outcomes. An event is a collection of one or more outcomes. A sample space is the set of all possible outcomes.

Probabilities can be written as decimals or fractions and are in the range of 0 (impossible) to 1 (certain)

If each outcome has an equal likelihood of occurring,

$$
\text { Probability of event }=\frac{\text { number of possible outcomes in the even }}{\text { total number of possible outcomes }}
$$

Example 1: The table shows the time taken, in minutes, for a group of students to complete a number puzzle.

| Time, $t(\mathrm{~min})$ | $5 \leq t<7$ | $7 \leq t<9$ | $9 \leq t<11$ | $11 \leq t<13$ | $13 \leq t<15$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 6 | 13 | 12 | 15 | 4 |

A student is chosen at random. Find the probability that they finished the number puzzle:
a. In under 9 minutes

Total number of students: $6+13+12+15+4=50$
Number of students who finished under 9 minutes: $6+13=19$
$P$ (finished under 9 minutes) $=\frac{19}{50}$
b. In over 10.5 minutes
10.5 minutes is $\frac{3}{4}$ through the $9 \leq t<11$ class. Estimate using interpolation
$\frac{1}{4} \times 12=3$
$3+15+4=22$
$P$ (finished in over 10.5 minutes) $=\frac{22}{50}$

## Venn Diagrams

A Venn diagram can be used to represent events graphically. Frequencies or probabilities can be placed in the regions of Venn diagrams.

A rectangle represents the sample space, $S$. It contains closed curves which represent events.


Union of A and shows the event in which either A or B or both occur


The shaded area shows the event in which A does not occur

## Mutually exclusive independent events

Events which have no outcomes in common are called mutually exclusive. The closed curves do not overlap in a Venn Diagram.

or mutually exclusive events,

$$
\mathrm{P}(A \text { or } B)=\mathrm{P}(A)+\mathrm{P}(B)
$$

When one event has no effect on another, they are independent. For independent events $A$ and $B$, the probability of $B$ happening is the same regardless of whether $A$ happens. or independent events,

$$
\mathrm{P}(A \text { and } B)=\mathrm{P}(A) \times \mathrm{P}(B)
$$

You can also use this multiplication rule to check if events are independent.
Example 2: The Venn diagram shows the number of students in a particular class who watch any of three popular TV shows

a. Find the probability of a student chosen at random watches $B$ or $C$ or both $4+5+10+7=26$
$\mathrm{P}($ watches $B$ or $C$ or both $)=\frac{26}{30}=\frac{13}{15}$
b. Determine whether watching $A$ and watching $B$ are statistically independent. $\mathrm{P}(A)=\frac{3+4}{30}=\frac{7}{30}$
$\mathrm{P}(B)=\frac{4+5+10}{30}=\frac{19}{30}$
$\mathrm{P}(A$ and $B)=\frac{4}{30}=\frac{2}{15}$
$P(A) \times P(B)=\frac{7}{30} \times \frac{19}{30}=\frac{133}{900}$
$\mathrm{P}(A$ and $B) \neq \mathrm{P}(A) \times \mathrm{P}(B)$
Therefore, watching $A$ and watching $B$ are not statistically independent

## Tree diagrams

A tree diagram can be used to show the outcomes of two or more event happening in succession

Example: A bag contains seven green beads and five blue beads. A bead taken from the bag at random and not replaced. A second bead is then taken from the bag. Find the probability that:
a. Both beads are green

1. Draw a tree diagram to show the events.

2. Multiply along the branch of tree diagram
$\mathrm{P}($ green and green $)=\frac{7}{12} \times \frac{6}{11}=\frac{7}{22}$
b. The beads are different colours
$P$ (different colours) $=P($ green then blue $)+P($ blue then green $)$

$$
\begin{aligned}
& =\frac{7}{12} \times \frac{5}{11}+\frac{5}{12} \times \frac{7}{11} \\
& =\frac{35}{66}
\end{aligned}
$$

